

7.3 Double Angle Formulas

Using the compound angle formulas you learned from 7.2, develop a formula for each of the following:

$$\sin(2\theta)$$

$$\cos(2\theta)$$

$$\tan(2\theta)$$

Double Angle Formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\begin{aligned} \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= 1 - 2 \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \end{aligned}$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Ex. 1 Express each of the following as a single trigonometric function.

a) $2 \sin 3x \cos 3x$

b) $\frac{2 \tan\left(\frac{y}{2}\right)}{1 - \tan^2\left(\frac{y}{2}\right)}$

c) $2 \cos^2(6x) - 1$

d) $5 - 10 \sin^2\left(\frac{x}{3}\right)$

Ex. 2 Use a double angle formula to rewrite each trigonometric ratio.

a) $\sin(4\theta)$

b) $\cos(5\theta)$

c) $\tan\left(\frac{\theta}{2}\right)$

Ex. 3 Determine the exact value of each expression.

a) $2\cos\left(\frac{\pi}{12}\right)\sin\left(\frac{\pi}{12}\right)$

b) $1 - 2\sin^2\left(\frac{5\pi}{8}\right)$

Ex. 4 Determine the exact value of each expression.

a) Suppose $\cos \theta = \frac{5}{\sqrt{22}}$, $\frac{3\pi}{2} < \theta < 2\pi$, determine $\sin(2\theta)$

b) Suppose $\tan \theta = \frac{4}{7}$, $\pi < \theta < \frac{3\pi}{2}$, determine $\cos(2\theta)$